MICROSTRUCTURAL RAYLEIGH WAVE DISPERSION ON A FLUID-COUPLED ANISOTROPIC SURFACE WITH VERTICAL LAMINATION

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INTRODUCTION

In a recent paper Nayfeh et al. [1] presented theoretical and experimental results for the propagation of longitudinal waves in a composite whose microstructure was large enough to cause observable velocity dispersion. Only wave propagation along the fiber axis of a uniaxial laminate was considered. A reflection coefficient was also derived for the case of normal incidence and parallel to the fibers. For ultrasonic inspection applications, what is required is the ability to analyze situations in which the wave is incident at arbitrary angles. Analysis of such general situations are, however, difficult to treat. A relatively simpler two-dimensional composite, which has been analyzed for an off-normal incident angle [2], consists of a bilaminated model with layers bonded and stacked normal to the x_3-direction. The structure occupies the half-space $x_2 \geq 0$ as illustrated in Fig. 1. The composite is immersed in water such that the x_2-direction is normal to the fluid-composite interface and the wave is incident from the fluid in the x_1-x_2 plane. For this model the reflection coefficient and the characteristic equation for the propagation of fluid-composite interfacial waves was calculated. The results reported in [2] are also restricted such that the individual composite components are isotropic.

In this paper we generalize the analysis of [2] to the case where the composite components are allowed to be anisotropic and possess as low as monoclinic symmetry. Experimental verification of the model is also conducted and included in the form of dispersion curve comparisons.

Analysis

Due to the symmetry of layering and loading we isolate from Fig. 1 the smallest repeating unit cell, which clearly consists of two half-laminates with the thicknesses h_1 and h_2 bonded at their interface. For

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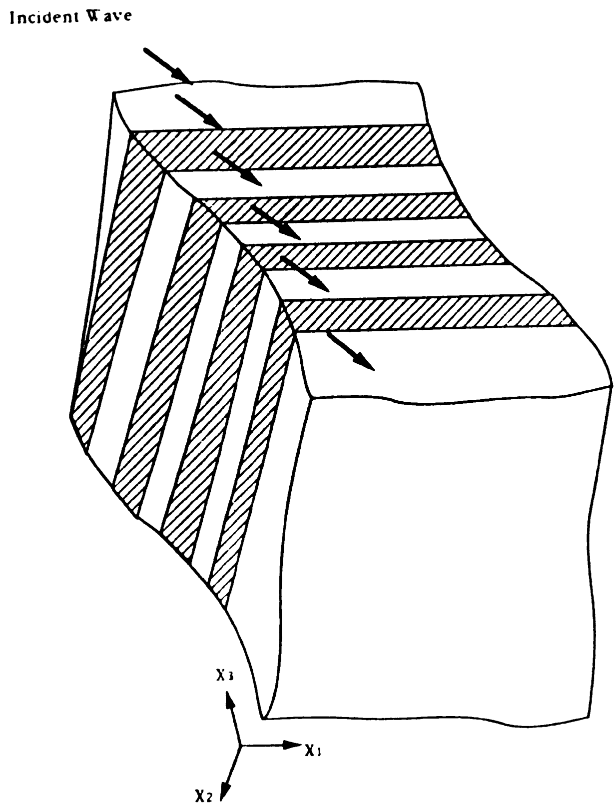

FIG. 1 COMPOSITE MODEL

the convenience analysis we choose a local coordinate $x_3^{(k)}$ for each laminate. (Here, k = 1,2 designates the two). In terms of the global coordinates x_i, i = 1, 2, and the local micro-coordinate x_3 we summarize the relevant field equations and associated interface and symmetry conditions for each material laminate k as

$$\frac{\partial \sigma_{\alpha\beta}}{\partial x_\beta} - \rho \frac{\partial_2 u_\alpha}{\partial t^2} = - \frac{\partial \sigma_{\alpha 3}}{\partial x_3} \tag{1}$$

The constitutive relations are:

$$\sigma_{\alpha\beta} = c_{\alpha\beta\gamma\delta} e_{\gamma\delta} + c_{\alpha\beta 33} e_{33} \tag{2}$$

$$\sigma_{33} = c_{33\gamma\delta} e_{\gamma\delta} + c_{3333} e_{33} \tag{3}$$

$$\sigma_{\alpha 3} = c_{\alpha 3\beta 3} e_{\beta 3} \tag{4}$$

with

$$u_\alpha^{(1)} = u_\alpha^{(2)} , \; \sigma_{\alpha 3}^{(1)} = \sigma_{\alpha 3}^{(2)} , \; u_3^{(1)} = u_3^{(2)} , \; \sigma_{33}^{(1)} = \sigma_{33}^{(2)} , \tag{5}$$

at the interface $x_3^{(1)} = h_1$ and $x_3^{(2)} = -h_2$, and

$$\sigma_{\alpha 3}^{(k)} = 0, \; u_3^{(k)} = 0 \; , \tag{6}$$

on the central planes $x_3^{(k)}$. In the above equations the usual tensorial summation convention holds.

Following the procedure of [2], Eqs. (1) and (2) are averaged across their respective thicknesses and the appropriate symmetry and continuity conditions (5) and (6) are utilized we get

$$n_k \rho_k \frac{\partial^2 \bar{u}_\alpha^{(k)}}{\partial t^2} - n_k \frac{\partial \bar{\sigma}_{\alpha\beta}^{(k)}}{\partial x_\beta} = (-1)^{k+1} P_\alpha \; , \quad \alpha, \beta = 1,2 \tag{7}$$

$$n_k \bar{\sigma}_{\alpha\beta}^{(k)} - n_k c_{\alpha\beta\gamma\delta}^{(k)} \bar{e}_{\gamma\delta}^{(k)} = (-1)^{\alpha+1} c_{\alpha\delta 33}^{(k)} \; S \tag{8}$$

where $n_k = (h_k/h)$, with $h = (h_1 + h_2)$ designates the volume fraction of material k; P_α and S are momentum and constitutive relation interaction terms given by

$$P_\alpha h = \sigma_{\alpha 3}^{(1)}(h_1) = \sigma_{\alpha 3}^{(2)}(-h_2) = -\sigma_{\alpha 3}^{(2)}(h_2) \tag{9a}$$

$$Sh = u_3^{(1)}(h_1) = u_3^{(2)}(-h_2) = -u_3^{(2)}(h_2) \; . \tag{9b}$$

The constitutive relations (3) and (4) will now be used to derive an approximate expression for such relations. To this end we first invert Eq. (4) and, guided by Eqs. (5) and (6) and by the definition of P_α in Eq. (9), expand $\sigma_{\alpha 3}^{(\alpha)}$ to first order

$$\sigma_{\alpha 3}^{(1)} = \frac{P_\alpha}{n_1} x_3^{(1)} \; , \qquad \sigma_{\alpha 3}^{(2)} = - \frac{P_\alpha}{n_2} x_3^{(2)} \; . \tag{10}$$

Substituting from (10) into (4) , multiplying the resulting equation by the appropriate $x_3^{(k)}$ and integrating across laminate thicknesses by parts, after neglecting $u_3^{(k)}$ and satisfying the interface continuity condition on u_3, yields (see [2])

$$P_1 = \frac{3}{h^2} \{ G_{11} (\bar{u}_2^{(2)} - \bar{u}_2^{(1)}) + G_{12} (\bar{u}_1^{(2)} - \bar{u}_1^{(1)}) \} \tag{11}$$

$$P_2 = \frac{3}{h^2} \{ G_{12} (\bar{u}_2^{(2)} - \bar{u}_2^{(1)}) + G_{22} (\bar{u}_1^{(2)} - \bar{u}_1^{(1)}) \} \tag{12}$$

where

$$s_{44} = \frac{c_{1313}}{c_{1313}c_{2323} - c_{2313}^2}, \quad s_{55} = \frac{c_{2323}}{c_{1313}c_{2323} - c_{2313}^2}$$

$$s_{45} = \frac{-c_{2313}}{c_{1313}c_{2323} - c_{2313}^2} \tag{13}$$

$$G_{11} = (s_{55}^{(1)} n_1 + s_{55}^{(2)} n_2)/\Delta, \quad G_{12} = (s_{45}^{(1)} n_1 + s_{45}^{(2)} n_2)/\Delta$$

$$G_{22} = (s_{44}^{(1)} n_1 + s_{44}^{(2)} n_2)/\Delta$$

$$\Delta = (s_{44}^{(1)} n_1 + s_{44}^{(2)} n_2)(s_{55}^{(1)} n_1 + s_{55}^{(2)} n_2) - (s_{45}^{(1)} n_1 + s_{45}^{(2)} n_2)^2 \tag{14}$$

If we average equation (3) for each material k and equate the results, we get

$$S = \frac{1}{E}[c_{33\gamma\delta}^{(2)} \bar{e}_{\gamma\delta}^{(2)} - c_{33\gamma\delta}^{(1)} \bar{e}_{\gamma\delta}^{(1)}] \tag{15}$$

where

$$E = \frac{c_{3333}^{(1)}}{n_1} + \frac{c_{3333}^{(2)}}{n_2} \tag{16}$$

In Eqs. (12) and (13), assuming their left hand sides stay finite while letting $h \to 0$, dictates that $\bar{u}_\alpha^{(1)} \to \bar{u}_\alpha^{(2)} = \bar{u}_\alpha$. For this limiting case (15) reduces to

$$S = \frac{1}{E}[c_{33\gamma\delta}^{(2)} - c_{33\gamma\delta}^{(1)}] \bar{e}_{\gamma\delta} \quad . \tag{17}$$

Substituting from (17) into (8) and again summing for $\alpha = 1, 2$ we get the effective two-dimensional constitutive relation

$$\bar{\sigma}_{\alpha\beta} = F_{\alpha\beta\gamma\delta} \bar{e}_{\gamma\delta} \tag{18}$$

where

$$F_{\alpha\beta\gamma\delta} = n_1 c_{\alpha\beta\gamma\delta}^{(1)} + n_2 c_{\alpha\beta\gamma\delta}^{(2)} + \frac{1}{E}\{c_{33\gamma\delta}^{(2)} - c_{33\gamma\delta}^{(1)}\}\{c_{\alpha\beta33}^{(1)} - c_{\alpha\beta33}^{(2)}\} \tag{19}$$

and

$$\bar{e}_{\gamma\delta} = \frac{1}{2}\left(\frac{\partial \bar{u}_\gamma}{\partial x_\delta} + \frac{\partial \bar{u}_\delta}{\partial x_\gamma}\right), \tag{20}$$

define effective mixture elastic constants and the two-dimensional strain tensor. This limit, (i.e., $h \to 0$) which can also be designated as the "strong coupling" limit is equivalent to the static limit of the laminated composite. Here the composite is replaced by a homogenized and nondispersive medium whose properties are weighted functions of the individual constituents.

Now we use the representative coupled mixture equations (7) and (8) to derive the reflection coefficient from a fluid loaded half-space of the laminated composite. The fact that we are using these coupled equations will lead to a dispersive behavior of the medium which otherwise is absent for homogeneous materials. To this end we substitute from (11), (12), (13) and (15) into (8) and (7) and note that P_α, S, and $\bar{e}^{(k)}_{\alpha\beta}$ are functions of the displacements $\bar{u}^{(k)}_\alpha$. This results in the four coupled equations

$$\rho_k n_k \frac{\partial^2 \bar{u}^{(k)}_\alpha}{\partial t^2} - [F^{(k)}_{\alpha\beta\gamma\delta}\, n_k \frac{\partial \bar{e}^{(k)}_{\gamma\delta}}{\partial x_\beta} + (-1)^{k+1}\{c^{(k)}_{\alpha\beta 33}\, \frac{\partial S}{\partial x_\alpha} + P_\alpha\}] = 0 \ , \quad (21)$$

($k = 1,2$ and $\alpha = 1,2$). For harmonic waves propagating in the x_1 direction these four equations admit the formal solutions

$$\bar{u}^{(k)}_\alpha = U^{(k)}_\alpha \exp\,[i\xi(x_1 + ct + \eta x_2)] \ , \quad (22)$$

where $U^{(k)}_\alpha$ are displacement amplitudes, ξ is the wave number, $c = \omega/\xi$ (with ω being the circular frequency) is the phase velocity and $\eta\xi$ is the x_2-direction component of the wave number. With reference to Eq. (22), for surface waves to exist, η must have a positive imaginary part. If (22) is substituted into (21) we get the characteristic equation relating η to c and ξ as

$$\begin{vmatrix} \varepsilon D_1 - 1 & 1 & \varepsilon T_1 - r_1 & r_1 \\ 1 & \varepsilon D_2 - 1 & r_1 & \varepsilon T_2 - r_1 \\ \varepsilon T_1 - r_1 & r_1 & \varepsilon Q_1 - r_2 & r_2 \\ r_1 & \varepsilon T_2 - r_1 & r_2 & \varepsilon Q_2 - r_2 \end{vmatrix} = 0 \quad (23)$$

where

$$\begin{aligned} D_k &= \rho_k n_k c^2 - F^{(k)}_{11} - 2F^{(k)}_{16}\,\eta - F^{(k)}_{66}\,\eta^2 \\ Q_k &= \rho_k n_k c^2 - F^{(k)}_{66} - 2F^{(k)}_{26}\,\eta - F^{(k)}_{22}\,\eta^2 \\ T_k &= -[F^{(k)}_{16} - (F^{(k)}_{12} + F^{(k)}_{66})\eta - F^{(k)}_{26}\,\eta^2] \\ r_1 &= G_{12}/G_{11} \ , \quad r_2 = G_{22}/G_{11} \\ \varepsilon &= h^2\xi^2/(3G_{11}) = h^2\omega^2/(3c^2 G_{11}) \ , \end{aligned} \quad (24)$$

In Eq. (24) we use the contracted index notation 11-1, 22-2, 12-6 to write out the various entries $F^{(k)}_{\alpha\beta\gamma\delta}$. Equation (23) admits four solutions for η^2 as compared with two solutions for homogeneous materials or for the homogenized composite. Only two of these roots assume boundedness of the solutions. Identifying these roots by η_1 and η_2 (their actual values will be obtained numerically) we can proceed to derive the required required reflection coefficient.

Using superposition, we now write (24) as

$$\bar{u}_\alpha = [U_{\alpha 1} e^{i\xi\eta_1 x_2} + U_{\alpha 2} e^{i\xi\eta_2 x_2}] e^{i\xi(x_1 + ct)} , \qquad (25)$$

Here we recognize that the entire effect of the microstructure (i.e., the influence of frequency) is contained in the expressions for η_1 and η_2. If we adopt such a passive role of the frequency then we can derive the required reflection coefficient for the homogenized medium. Invoking the standard fluid-solid boundary conditions at $x_2 = 0$ yields the reflection coefficient

$$R = (Z_1 - Z_2)/(Z_1 + Z_2) , \qquad (26)$$

$$Z_1 = Z_{11}Z_{22} - Z_{12}Z_{21} \qquad (27a)$$

$$Z_2 = \frac{\rho_f c^2}{y_f^2} (W_1 Z_{22} - W_2 Z_{21}) \qquad (27b)$$

where

$$Z_{1\alpha} = F_{12} + F_{22}\eta_\alpha W_\alpha + F_{26} (\eta_\alpha + W_\alpha)$$

$$Z_{2\alpha} = F_{16} + F_{26}\eta_\alpha W_\alpha + F_{66} (\eta_\alpha + W_\alpha)$$

$$W_\alpha = \frac{[\rho_c c^2 - F_{11} - 2F_{16}\eta_\alpha - F_{66}\eta_\alpha^2]}{[F_{16} + (F_{12} + F_{66})\eta_\alpha + F_{26}\eta_\alpha^2]} \qquad (28)$$

$$\eta_f^2 = \frac{c^2}{c_f^2} - 1 , \qquad (29)$$

and ρ_f is the fluid density.

RESULTS AND DISCUSSION

The sample utilized in these studies is a stack of copper and stainless steel plates (elastic properties collected in ref. 3). By maintaining a large static clamping force on the stack, the welded contact boundary conditions for both stresses and displacements could be satisfied. The wave propagation surface has been carefully polished to a mirror finish to minimize any perturbing influences arising from surface roughness. The sample used for most of the measurements has a unit cell dimension of 0.79 mm.

Measurements are performed by insonifying the sample with an ultrasonic beam from a conventional piston transducer at selected angles. These angles are related to a trace velocity through Snell's law. When the trace velocity equals the phase velocity of a dispersive Rayleigh wave, strong mode coupling will deform and displace the beam energy in the reflected field. Away from this condition, the reflection will be essentially specular. To study Rayleigh wave dispersion, data are acquired in a high-resolution ultrasonic scanning system by exciting the broadband transducer with rf tone bursts. The reflected acoustic field is detected by a second transducer positioned at the negative incident angle. Spectra are accumulated by stepping the tone-burst frequency and observing the receiver signal. Typically, the spectra show a pronounced minimum at the frequency where strong mode conversion occurs. This fact permits us to detect the dispersive Rayleigh wave and measure its phase velocity very accurately, without performing the more difficult time-of-flight measurement.

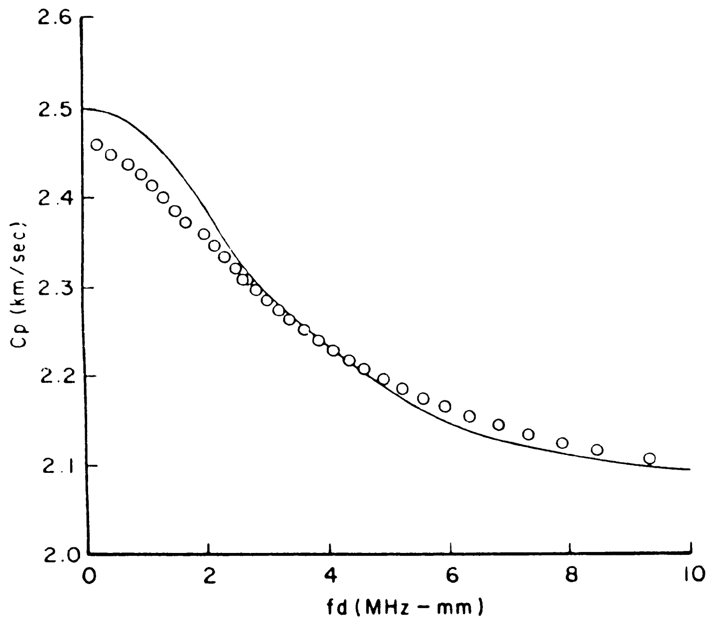

FIG. 2 THEORETICAL AND EXPERIMENTAL COMPARISONS OF SURFACE VELOCITY DISPERSION.

Results

Repeating such experiments at many incident angles and recording the trace velocity and minimum frequency allows us to construct a dispersion curve for Rayleigh waves on the edge-laminated plate. The results of such measurements are displayed in Fig. 2. The Rayleigh wave phase velocity is plotted on the ordinate as a function of frequency times the unit cell dimension. The solid curve is the calculation obtained from the vanishing of the denominator of the reflection coefficient (26) and the open circles are the experimental measurements. With the possible exception of the low frequency limit the agreement between the two is very good, especially considering the approximations necessary to arrive at the predicted behavior. Even in the quasistatic limit, the disparity is only 1.5%. At low frequency the Rayleigh velocity tends, as expected, to the mixture value of the two constituents. At high frequency the velocity approaches the surface wavespeed of the more compliant of the constituents. This behavior is very well rendered by the model.

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